This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

War
4299
03

War 4299.03

Harvard College Library

FROM

The Author

Harvard University, Cambridge U.S.A.
from the author.

Phys 1066.5

A HISTORICAL SKETCH

OF THE

EXPERIMENTAL DETERMINATION

OF THE

RESISTANCE OF THE AIR TO THE MOTION OF PROJECTILES

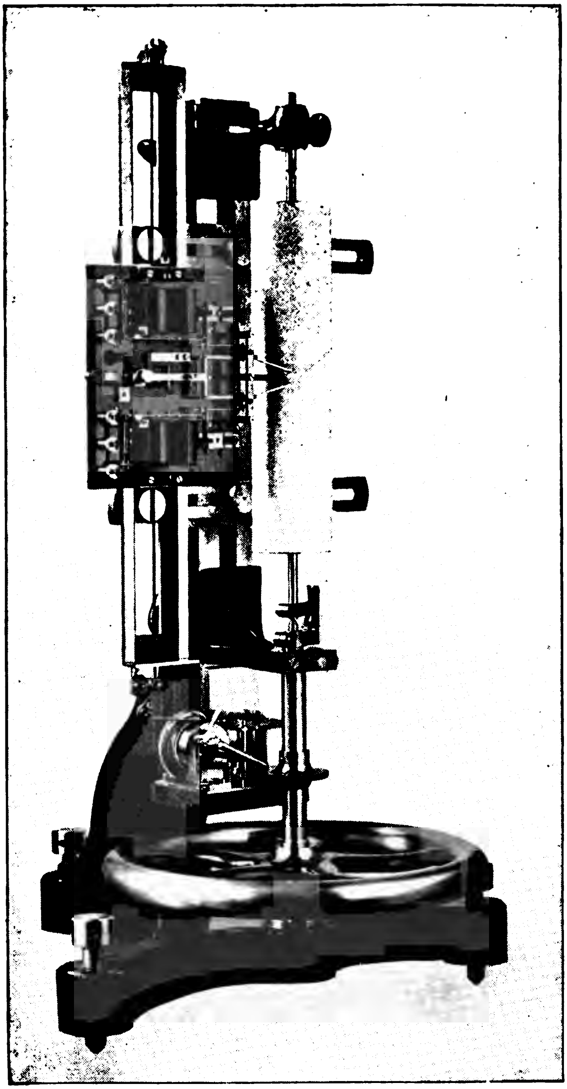
BY

FRANCIS BASHFORTH, B.D.

LATE PROFESSOR OF APPLIED MATHEMATICS TO THE ADVANCED CLASS OF
ROYAL ARTILLERY OFFICERS, WOOLWICH, AND
FORMERLY FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1903

[*All Rights reserved.*]



A HISTORICAL SKETCH
OF THE
EXPERIMENTAL DETERMINATION
OF THE
RESISTANCE OF THE AIR TO THE MOTION OF PROJECTILES

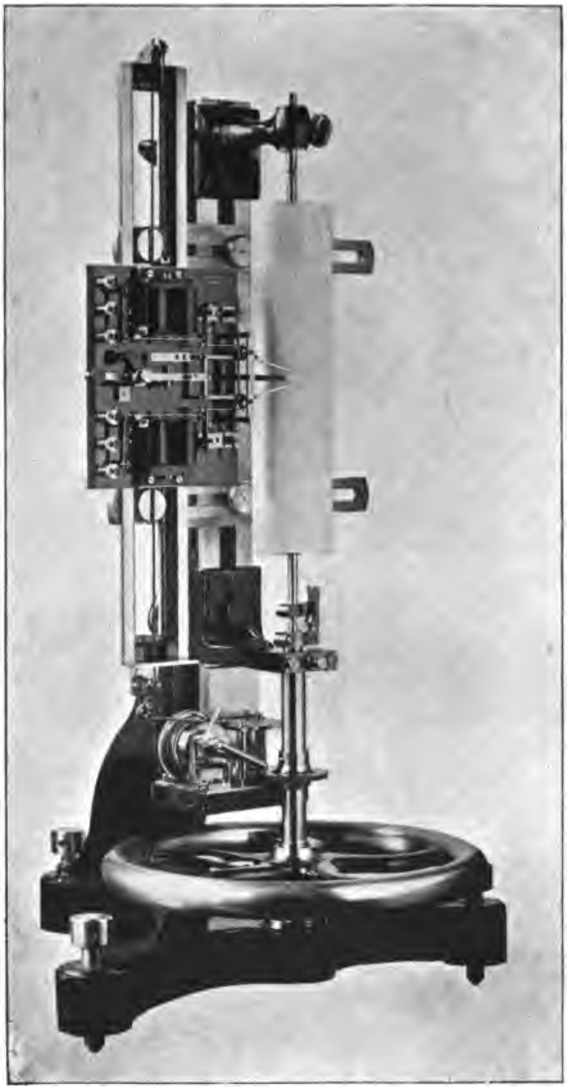
BY

FRANCIS BASHFORTH, B.D.

LATE PROFESSOR OF APPLIED MATHEMATICS TO THE ADVANCED CLASS OF
ROYAL ARTILLERY OFFICERS, WOOLWICH, AND
FORMERLY FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1903

[*All Rights reserved.*]



A HISTORICAL SKETCH
OF THE
EXPERIMENTAL DETERMINATION
OF THE
RESISTANCE OF THE AIR TO THE MOTION OF PROJECTILES

BY

FRANCIS BASHFORTH, B.D.
LATE PROFESSOR OF APPLIED MATHEMATICS TO THE ADVANCED CLASS OF
ROYAL ARTILLERY OFFICERS, WOOLWICH, AND
FORMERLY FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.
1903

[*All Rights reserved.*]

~~Plays 1066.5~~
War 42.19.03

HARVARD COLLEGE LIBRARY
AUG 31 1903
CAMBRIDGE, MASS.

The Author.

London: C. J. CLAY AND SONS,
CAMBRIDGE UNIVERSITY PRESS WAREHOUSE,
AVE MARIA LANE.
Glasgow: 50, WELLINGTON STREET.

Leipzig: F. A. BROCKHAUS.
New York: THE MACMILLAN COMPANY.
Bombay and Calcutta: MACMILLAN AND CO., LTD.

Cambridge:
PRINTED BY J. AND C. F. CLAY,
AT THE UNIVERSITY PRESS.

RESISTANCE OF THE AIR TO THE MOTION OF PROJECTILES.

[1600] GALILEO and most early writers on ballistics assumed that the resistance of the air to projectiles produced little or no effect on their motion. Robins remarks that “if it were necessary, a large “catalogue of geometers of note might be here produced, who “have asserted in their works, that, in the operation of gunnery, “the resistance of the air was too minute to merit attention. “And I do not remember, that any author has formally or “expressly contradicted this position; whatever may in general “be concluded, from what some of them have at times advanced “as to the laws observed by resisted bodies. Indeed I may venture to affirm, that it is now the almost universal opinion of those, “who have studied the doctrine of projectiles from the treatises, “which have hitherto been published thereon: that all shells and “bullets in their flight do nearly describe the curve of a parabola; “and consequently, that the resistance of the air to the motion of “these bodies is altogether inconsiderable.” *Tracts*, I. 176.

[1742] Robins appears to have carried out his ballistic experiments chiefly at his own expense. His first ballistic pendulum weighed 56 pounds, afterwards increased to 97 pounds. Robins arrived at the conclusion that for a velocity less than 1100 f.s. and greater than 1200 f.s., the resistance of the air to spherical projectiles will vary as the square of the velocity, but that the “absolute quantity “of that resistance in these greater velocities will be near three “times as great, as it should be by comparison with the smaller “velocities.” *Tracts*, I. 182. My experiments shew that the law of resistance to spherical projectiles varies approximately as the square of the velocity for velocities less than 840 and greater than 1300 f.s., where the coefficient for high velocities is nearly double that for low velocities.

The Royal Society awarded their Copley Medal to Robins in 1746. His experiments and theories are said to have met with the greatest approbation from the best judges at home. His work was translated by Euler into High Dutch, 1745, and by Le Roy into French, 1751. He was appointed engineer-general to the East India Company, 1749, for which he was to receive £500 a year for life, provided he gave his services for five years in India.

Two years after Hutton arrived at Woolwich, 1775, he "in [1775]
"conjunction with some able officers of the Royal Regiment of "Artillery, and other ingenious gentlemen, first instituted a course "of experiments on fired gunpowder and cannon balls" for government. His "account of them was presented to the Royal "Society, who honoured it with the gift of the annual gold medal, "and printed it in the *Philosophical Transactions* for the year "1778."

Hutton remarks "That part of Mr Robins's book has always "been much admired, which relates to the experimental method "of ascertaining the actual velocities of shot, and in imitation of "which, but on a *large scale*, those experiments were made which "were described in my paper." The projectiles used by Hutton varied from one to near three pounds in weight, afterwards increased to six pounds. Further experiments were carried on from 1783 to 1791, and the results obtained were considered satisfactory. Hutton at first made use of a pendulum weighing about 600 lbs. It was increased to 1014 lbs. in 1788, afterwards to 1655 lbs., and at last to 2099 lbs. Hutton's health began to fail in 1806, and when he retired the Board of Ordnance manifested their approbation of his long and meritorious services by granting him a pension for life of £500 per annum.

Gregory made some experiments at Woolwich in 1815–18 with a ballistic pendulum weighing 7046 lbs. chiefly on the effect of windage.

The first experiments made with the ballistic pendulum at [1839]
Metz were carried out during 1839, 1840. The weights of the projectiles were about 9, 13, 26 and 51 lbs. The weight of the pendulum was 13,228 lbs. Other experiments were made at

Metz 1856, 7 and 8. The experiments made with the electro-ballistic pendulum, "n'ont pas donné une formule qu'on puisse "appliquer au tir avec une entière confiance." Didion, *Traité* &c., p. 71, 1860. The final result arrived at by Didion (*Lois* &c., 1857, p. 79) was $\rho = 0{\cdot}027\pi R^2V^2(1+0{\cdot}0023V)$, where V is the velocity, R the radius of the spherical projectile, and ρ the resistance of the air; the metre, the second and the kilogramme being the units. For English measures, the resistance in pounds is

$$0{\cdot}0000028d^2v^2(1+0{\cdot}0007v),$$

where d is the diameter in inches and v the velocity in f.s. of the shot.

Didion remarks, "La recherche des lois de la résistance que "les fluides opposent au mouvement des corps solides présente "de grandes difficultés, tant sous le point de vue mathematique "que sous celui des expériences, à cause de la complication du "phénomène. Aussi, malgré les travaux des plus célèbres géo-"mètres, malgré les nombreuses expériences qui ont été faites, "anciennement ou récemment, la question est loin d'être résolue "d'une manière générale, surtout en ce qui touche la forme des "corps; mais, pour les mobiles de form *sphérique* employés par "l'artillerie la question est beaucoup plus avancée." Didion, *Traité* &c., 1860, p. 27.

The Ballistic officer of the Ordnance Select Committee reported to them, that "It is to be regretted that this subject "cannot be fully treated in the present report, but the *difficulties* "*in the way of a clear solution are so many, and so great,* that it "would be difficult, *with our present experience, to assign any new* "*law representing, with accuracy,* the RESISTANCE OF THE AIR TO "THE MOTION OF SPHERICAL AND ELONGATED PROJECTILES," p. 19, 1865.

Hutton has given a table of the resistances of the air to a ball two inches in diameter for velocities 100 to 2000 f.s. which is remarkably good. *Tracts*, III. 230. I have also calculated the corresponding resistances from the Metz formula for comparison with my own results for spherical projectiles, 1870.

These experiments were carried out with great success by the use of the ballistic pendulum. Hutton's results were better than the French. And I should in general give the preference for accuracy to the results obtained by the use of a light pendulum. But the ambition of experimenters has been to make use of heavy projectiles suitable for warlike purposes, in which case a heavy pendulum became a necessity. Each succeeding experimenter increased the weight of his pendulum till the Woolwich pendulum was reached in 1855.

Vel. f.s.	Hutton, 1791 lbs.	Diff. lbs.		Didion, 1857 lbs.	Diff. lbs.		Bashforth, 1870 lbs.
900	17·5	-5·0	=	14·8	-2·3	=	12·5
1000	22·6	-5·1	=	19·0	-1·5	=	17·5
1100	28·6	-3·6	=	24·0	+1·0	=	25·0
1200	35·3	-2·4	=	29·7	+3·2	=	32·9
1300	42·7	-2·4	=	36·2	+4·1	=	40·3
1400	50·7	-2·5	=	43·5	+4·7	=	48·2
1500	59·2	-3·0	=	51·7	+4·5	=	56·2
1600	67·9	-3·0	=	60·8	+4·1	=	64·9
1700	76·8	-3·1	=	70·9	+2·8	=	73·7
1800	85·5	-3·5	=	82·0	0	=	82·0
1900	94·1	-1·5	=	94·2	-1·6	=	92·6
2000	102·4	+0·9	=	107·5	-4·2	=	103·3

Mention is sometimes made of electro-ballistic instruments, but I am not acquainted with results of any value obtained by the sole use of them.

Konstantinoff was the first to propose to obtain numerous records for each round fired. But he seems to have had no notion how these desired results were to be arrived at, for in 1843, he employed M. Bréguet of Paris, to invent and construct the required instrument for him. But this was a failure. Moigno, *Télégraphie*, p. 106.

On the institution of the Advanced Class of Royal Artillery [1864]
Officers, 1864, the Council of Military Education invited me to undertake the Professorship of Applied Mathematics. I accepted that office, as it then appeared to me possible to obtain a satisfactory solution of the problem of the resistance of the air to the motion of projectiles, and also that this work might be completed in two years, supposing that reasonable assistance was afforded me in the small demands I should have to make upon the govern-

ment. But the President and Vice-President of the Ordnance Select Committee were opposed to everything I proposed to do. They professed to know all about ballistics. They had their Navez E.B. instrument, which was, *at that time*, everything they could desire. Chronographs with rotating cylinders had been tried and had failed. They had made all necessary experiments, and were prepared to furnish me with any amount of results from their own stores!

[1864] I. It now became evident that if anything was to be done I must do it myself. I therefore prepared a rough lithographic sketch of the proposed new chronograph. This plan, accompanied by a letter to the Director of Artillery Studies, was forwarded to the War Office, because, unless I could depend upon having permission to use my chronograph, it was useless for me to proceed further in the matter. I received a favourable reply from the Director of Ordnance, May 17, 1864, concluding, "I am to "add that Earl de Gray and Ripon recognises with much pleasure "the zealous and practical manner in which you have entered "upon your duties." The construction of the new chronograph was therefore commenced by me in August 1864, and in the following June 1865 it was reported ready for trial with *ten* screens. It received its first partial trial with only *two* screens, at some experiments made by a gun-cotton committee, when the President of the Royal Society was present, July 18 and 19, 1865. But the trial with *ten* screens did not commence till Nov. 23,
[1865] 1865, when rounds 1, 2, 5 good, and 3, 4 bad were fired in Plumstead Marshes. The screens were not quite satisfactory, Nov. 29, when rounds 7 good, and 6, 8, 9 bad were fired. A new set of screens was then provided, and on Dec. 12, rounds 10, 11, 13, 15, 16, 17, 18 good, and 12, 14 bad were fired with a satisfactory result. The report of the testing of the chronograph was forwarded Dec. 18, 1865, to the Council of Military Education, who at once ordered it to be printed. Reprinted in *Reports* &c. (1865–1870) 84/B/1941, pp. 3—9.

[1866] A description of the new chronograph was given in the *Proceedings of the Royal Artillery Institution*, Woolwich 1866, Vol. v., pp. 161—192. The observations made at the testing of the instrument were there expanded, with a view to exemplify its great practical utility. These results, printed in 1866, shall be now used to shew the value of the new invention. The following table, there given, shews the time, expressed to four places of the decimal of a second, at which the projectile of each good round passed the several screens.

Screen	1	2	3	4	5	6	7	8	9	10
Round	″	″	″	″	″	″	″	″	″	″
1	0·0	0·1064	0·2141	0·3229	0·4328	0·5437	0·6556	0·7682	0·8814	0·9950
2	·0	·1045	·2098	·3160	·4232	·5319	·6422	·7538	·8670	·9818
5	·0	·1046	·2102	·3170	·4248	·5338	·6438	·7553	·8682	*
7	·0	·1033	·2087	·3157	·4240	·5333	·6433	·7541	·8659	·9787
10	·0	·1054	·2117	·3189	·4273	·5368	·6477	·7600	·8736	·9884
11	·0	·1047	·2110	·3188	·4281	·5387	·6505	·7630	*	*
13	·0	·1051	·2112	·3183	·4266	·5362	·6471	·7592	·8723	·9866
15	·0	·1042	·2102	·3175	·4262	·5359	·6466	·7580	*	*
16	·0	·1050	·2111	·3185	·4273	·5375	·6490	·7617	·8757	·9908
17	·0	·1049	·2114	·3193	·4285	·5389	·6505	·7636	·8783	·9947
18	·0	·1057	·2123	·3200	·4287	·5385	·6495	·7618	·8754	·9905
Totals...	·0	1·1538	2·3217	3·5029	4·6975	5·9052	7·1258	8·3587	7·8578	7·9065
Divisors	...	11	11	11	11	11	11	11	9	8
Means...	·0	·10489	·21106	·31845	·42705	·53684	·64780	·75988	·87309	·98831

Distance from gun. ft.	Screen.	Average Time.	Δ_1	Δ_2
		″		
120	1	0·00000		
			+10489	
240	2	·10489		+128
			+10617	
360	3	·21106		+122
			+10739	
480	4	·31845		+121
			+10860	
600	5	·42705		+119
			+10979	
720	6	·53684		+117
			+11096	
840	7	·64780		+112
			+11208	
960	8	·75988		+113
			+11321	
1080	9	·87309		+201
			+11522	
1200	10	·98831		

The irregularity in the 10th screen is due to the failure of three rounds to reach that screen.

In 1867–8 systematic experiments were made with guns of 3, 5, 7 and 9-inch bores to determine the resistance of the air to the motion of projectiles. These experiments were all perfectly independent of the above mentioned *testing* experiments, and they were used in the calculation of the General Tables $S_v = S_V - sd^2/w$ and $T_v = T_V - td^2/w$, which we will now use to calculate the testing rounds. Here $d = 3$ inches; $w = 11$ lbs. 9 oz. = 11·5625 lbs.; $w/d^2 = 1·28472$; $d^2/w = 0·7783797$; $V = 1150·13$* velocity at the first screen, 120 feet from the gun. $S_V = 41261·884$, $S_v = S_V - n \times 120\, d^2/w$ $= 41261·884 - n \times 93·4056$ where n must be taken 0, 1, 2 ... 9 in succession.

Screen	Distance	S_v	v	T_v	$T_V - T_v$	t calc.
	feet		f. s.			
1	120	41261·884	1150·13	230″·8553	0·0000	0″·00000
2	240	41168·478	1136·75	·7734	0·0819	0·10522
3	360	41075·073	1123·70	·6909	0·1644	0·21121
4	480	40981·667	1111·01	·6072	0·2481	0·31874
5	600	40888·262	1098·75	·5225	0·3328	0·42755
6	720	40794·856	1087·01	·4371	0·4182	0·53727
7	840	40701·451	1075·84	·3507	0·5046	0·64827
8	960	40608·045	1065·29	·2634	0·5919	0·76043
9	1080	40514·640	1055·45	·1753	0·6800	0·87361
10	1200	40421·234	1046·44	·0865	0·7688	0·98769

The first and second rounds fired as *tests* of the new chronograph were very good. As the muzzle velocity for the first round fired is generally below the average, we will compare the mean times of the first *two test* rounds with above *calculated* times of passing each screen.

Screen	1	2	3	4	5
Rounds 1 and 2	0″·0	·10545	·21195	·31945	·42800
Calc. 1902	0·0	·10522	·21121	·31874	·42755
Difference	·0	+·00023	+·00074	+·00071	+·00045

Screen	6	7	8	9	10
Rounds 1 and 2	·53780	·64890	·76100	·87420	·98840
Calc. 1902	·53727	·64827	·76043	·87361	·98769
Difference	+·00053	+·00063	+·00057	+·00059	+·00071

We will now find the difference between the means for the eleven rounds of the observed *times* of passing each screen in the

---

* **Tables given in the *Woolwich Text Book of Gunnery*, 1902, have been used.**

test experiments 1865 with the times *calculated* by the tables of 1902.

Screen	1	2	3	4	5
	″				
Tests 1865	0·0	·10489	·21106	·31845	·42705
Calc. 1902	0·0	·10522	·21121	·31874	·42755
Difference	·0	−·00033	−·00015	−·00029	−·00050

Screen	6	7	8	9	10
Tests 1865	·53684	·64780	·75988	·87309	·98831
Calc. 1902	·53727	·64827	·76043	·87361	·98769
Difference	−·00043	−·00047	−·00055	−·00052	+·00062

Compare the above calculated velocities at which the projectile passed each screen with the average mean velocities calculated and published, 1866.

Screen	1	2	3	4
Calc. 1902	1150·13	1136·75	1123·70	1111·01
Published 1866		1136·8	1123·6	1110·9
Difference		−0·05	+0·10	+0·11

Screen	5	6	7	8
Calc. 1902	1098·75	1087·01	1075·84	1065·29
Published 1866	1098·7	1087·1	1075·9	1065·2
Difference	+·05	−0·09	−0·06	+0·09

Again, calculate the *velocities* at several points for comparison with those found from the *testing* experiments published 1866, at distances 130, 140, 150...990, 1000 feet from the gun.

Distance from gun.	S_v	Calculated v 1902	Difference		Exper. v 1865
feet.		f. s.	f. s.		f. s.
120	41261·884	...	...		...
130	41254·100	1149·0	+0·4	=	1149·4
140	41246·316	1147.9	+0·3	=	1148·2
150	41238·532	1146·7	+0·4	=	1147·1
200	41199·613	1141·2	+0·1	=	1141·3
300	41121·775	1130·2	0·0	=	1130·2
400	41043·937	1119·4	0·0	=	1119·4
500	40966·099	1108·9	−0·1	=	1108·8
600	40888·261	1098·8	−0·1	=	1098·7
700	40810·423	1088·9	+0·1	=	1089·0
800	40732·585	1079·5	+0·1	=	1079·6
900	40654·747	1070·5	0·0	=	1070·5
1000	40576·909	1061·9	−0·2	=	1061·7

Here again, we have an almost perfect agreement between calculated and experimental results.

I had, from the first, full confidence in the accuracy of the results furnished by my new chronograph. But considering the unfavourable circumstances under which my *testing* experiments were carried out in Plumstead Marshes Nov. and Dec. 1865, it was a great surprise to me to find that the results, then obtained, agreed so closely with those given by calculation founded on numerous systematic experiments *afterwards* carried out by numerous projectiles of various sizes, in 1867, 8. The calculations are given above in full, so that the errors, if any, may be easily detected. The minute agreement between perfectly independent experiments, here obtained, is a conclusive proof of the accuracy,

(1) of the *testing* experiments of 1865,

(2) of the *systematic* experiments of 1867, 8, and

(3) of the *general tables*, S_v and T_v, founded on experiments 1867, 8 and used to compare the results of (1) and (2).

Hence it appears that the chronograph is *perfect* for all practical ballistic purposes.

(4) My general tables S_v and T_v have been used, with satisfactory results to calculate the *time* of *flight* for the *lower elevations* given in range tables, p. 19 and (340) of second supplement.

(5) Making due allowance for jump and kite-like action, my Bernoulli Tables have been applied, with complete success, to calculate the *ranges* and *times* of flight for the *higher* elevations given in the range table of the 4-inch B.L. gun (344)—(358) second supplement.

Hence it appears that from first to last my chronograph has proved a perfectly satisfactory instrument in all ballistic experiments in which it has been employed.

1866] II. In Feb. 1866, it was arranged that experiments should be made with the new chronograph to determine the resistance of the air to elongated projectiles provided with (1) spherical heads;

(2) spheroidal heads; (3) ogival heads struck with *one* diameter; and (4) ogival heads struck with *two* diameters. The time appointed for the experiments to be carried out was Sep. 25—28. The frequent passage of ships during the experiments very much interfered with the firing of the gun, and on this account, not a single round could be safely fired on the 28th. There were only 44 rounds fired out of 70 prepared for the experiments. All the 20 hollow ogival projectiles were fired which gave 19 good rounds. As an indication of the rapidity attained in working the new chronograph, it was stated in the report that *nine* rounds, 23 to 31, giving 89 good records, were fired in 44′ 50″. It was also stated that "as the instrument is self-recording, the original "records of the above experiment can be seen by anyone, or they "can be submitted to any competent authority for examination or "recalculation," p. 17. This report was submitted by the War Office to the Ordnance Select Committee. They did their best to bring discredit upon my work, but as a copy of their report was forwarded to me, an opportunity was thus afforded me of making needful corrections and explanations. A second and amended report was made afterwards by the Ordnance Select Committee. *Reports* &c., pp. 10—17 and *Trans. of the Royal Society* 1868, pp. 417—441.

The experiments hitherto made were preliminary, and in each case were carried out with a fixed charge. The *first* set was used simply to test the chronograph. The *second* set was used to find resistance of the air to a few common forms of heads of elongated projectiles.

III. The real experiments remaining to be made were [186
intended to determine (1) the resistance of the air to *one form* of head of elongated projectiles for all useful velocities, and also (2) to find the effect produced by a variation of the diameter of the projectile. I was assisted in this case by the Director of Artillery Studies, the Professor of Artillery, R.M.A., and two R.A. officers, who had passed the Advanced Class. The military authorities selected for the experiment an ogival head ($1\frac{1}{2} d$), which was a very good form. The bores of the M.L. guns chosen were 3, 5, 7 and 9 inches. Captain Majendie, R.A. has stated that

"breech-loaders have been carefully compared for accuracy with "muzzle-loaders and the result has invariably been favourable to "the latter." *Proc. of the R.A. Inst.* 1870, p. 87. The charges were varied in order to obtain the required velocities, 900 to 1700 f.s. About 213 rounds of *elongated* projectiles were fired in fifteen days between Oct. 7, 1867 and May 21, 1868. *Reports* 84/B/1941, July 23, 1868, pp. 18—54. In consequence of the failure of the Ordnance Select Committee to give a satisfactory account of my former report of experiments, the Committee with whom I acted suggested that our report should "be referred to *mathematicians* "of eminence, such as the Astronomer Royal; Professor Adams, "Director of the Cambridge Observatory, *or* Professor Stokes, "Secretary to the Royal Society." About this time the Ordnance Select Committee was dissolved, when its president became Director General of Ordnance. The reference of our report was made by him, not to *one* of the three, as suggested, but to all the three, together with a representative of his own choice. *Reports* &c., Dec. 16, 1868, p. 153.

The experiments with *spherical* projectiles were afterwards carried out by the use of the same guns. By Nov. 6, 1868, 198
.869] rounds had been fired, which were reported Feb. 13, 1869, pp. 55—122. In this report the coefficients of resistance to spherical projectiles were given for velocities 850 to 2150 f.s. and have not been varied since. Here we remarked respecting our late report concerning *elongated* projectiles. "In order however to obtain a "more satisfactory table of values of $2000b \times w/d^2$ we have com"menced the recalculation of the times of passing each screen to "*five* places of decimals of a second," p. 65. This appendix was printed March 9, 1869, and referred with the other reports, pp. 123—152. These last coefficients for velocities 900 to 1700 f.s. have been commonly used. This was the completion of my experiments first published in 1870.

Sir G. B. Airy took the trouble to pay a visit to Shoeburyness to see my chronograph in action, but afterwards finding that he had not leisure to look to the records of past experiments he desired to retire from the Commission. He however expressed a decided opinion, saying, "But on the result of this I have *no* "*doubt whatever* that the Bashforth instrument is *far superior* to "the others," *Reports*, &c., p. 154.

The *Report* of the Referees, dated April 18, 1870, is a most [187 valuable document, wherein the merits of the various chronographs then used for ballistic purposes were fully discussed. These were divided by the Referees into three classes. (1) The Navez, Leurs, Benton and Vignotti instruments, in which the pendulum principle is employed to measure time. (2) The Boulengé instrument, in which the time is measured by the space passed over by a falling weight. (3) The Schultz instrument, "in which the time is "measured by the arc registered by two induction sparks on a "revolving cylinder, which arc is converted into time by means of "the undulating mark registered on the cylinder by a fork "vibrating so as to give a musical note of known pitch," p. 155.

"The above instruments, with the exception of that of Captain "Schultz, are adapted to measure only a single interval of time; "that of Captain Schultz alone is able to measure any number of "consecutive intervals," p. 155.

"The experiments recorded in Professor Bashforth's description "of his chronograph, as well as those given in his paper published "in the second part of the *Philosophical Transactions* for 1868, "afford ample materials for judging of the regularity of the indications of his instrument, and show that this regularity is everything that can be desired," p. 156.

"We have *not* been able to obtain experiments for *testing* in a "similar way the degree of constancy in the indications of the "Schultz instrument," p. 156.

"Now it is to be remarked that this action of the electro-"magnets on the vibrating fork constitutes a disturbing force "which interferes with the perfect uniformity of its vibrations. "In experiments described by Colonel Benét the number of double "vibrations per second was found to vary from 246 to 253. The "number of vibrations also varies in consecutive seconds in such "a way as to show that the period of the disturbances is much "less than a second, and that therefore during different parts of "the same second the fork may be vibrating at sensibly different "rates," p. 158.

"For this reason the time measures made by this instrument "cannot be regarded as at all commensurate with the extreme "delicacy of the indications of the instrument. In Professor "Bashforth's Chronograph the time is directly measured by means "of a clock with a half-seconds pendulum. Every double vibra-"tion of the pendulum is recorded on the revolving cylinder by "the breaking of the *same* galvanic current under exactly the "*same* conditions, and the time of the cannon balls passing "through each of the screens is recorded on the *same* revolving "cylinder by the momentary interruption of a second galvanic "current under exactly the *same* conditions," p. 158.

"Thus the time scale required for converting the records made "by this instrument into time intervals is here *supplied by the* "*instrument itself*, and consequently the results obtained by it are "unaffected by any error of the same kind as those to which we "have seen the former instruments to be liable. This freedom "from constant errors and from errors of reduction is one of the "great advantages of Professor Bashforth's instrument," p. 158.

"Next we have to consider the determination of the velocities "at several points of the path of the same projectile," p. 158.

"We have already stated that Schultz's instrument is the "only one which competes in this respect with that of Professor "Bashforth," p. 158.

"The objection that the rate of vibration of the fork in "Schultz's instrument is to a certain extent variable, probably on "account of the action of the electro-magnets, applies in this "case with its full force, for since the period of disturbance in the "rate of vibration is very short, it might very well happen that "the velocity near the beginning of the flight of the projectile "might be measured when the fork was vibrating most rapidly, "and the velocity near the end of the flight might be measured "when the fork was vibrating least rapidly, or *vice versâ*, and "thus the *change* of velocity, or the measure of the resistance of "the air, might be affected by a double error," p. 158.

"For this reason we consider that measures of the resistance "obtained by means of this instrument would be less trustworthy "than those found by that of Professor Bashforth," p. 158.

"We are therefore decidedly of opinion that, for the purpose "of determining several velocities of the same projectile and "thence deducing the resistance of the air, Professor Bashforth's "instrument is to be preferred to the other," p. 158.

"We consider that these experiments of Professor Bashforth "are admirably planned, and that the results obtained are very "valuable," p. 160.

The concluding question was—"Whether any means of solving "the same problem with equal precision existed before"? and the answer was—"We do not think that any means existed before of 'recording a number of successive small intervals of time with the "degree of precision and trustworthiness attained by Professor "Bashforth's instrument," p. 161.

A few additional particulars may be given respecting the Schultz instrument, which was exhibited at the conversazione given at the Conservatoire des Arts et Métiers, as well as at the Paris Exhibition, 1867. The Editor of the *Practical Mechanic's Journal* informs us that "Captain Schultz, in fact, finds that he "can observe and register time to $\frac{1}{10,000,000}$ of a second"! 1867, p. 195. Colonel Benét employed this instrument, of which he has published an account in his *Electro-Ballistic Machines*, 1866, but I am not informed that he ever used it with more than *two* screens, and thus he could measure only a *single* velocity by *one* instrument for each round fired.

A Schultz instrument was procured for the English Government, as was said, for the purpose of determining the internal pressure of fired gunpowder! "Various instruments have been lately "devised for this purpose, the *most perfect* of these being the "Chronoscope invented by Captain Schultz of the French Artillery "and the instrument just completed by Captain Andrew Noble "for the Ordnance Select Committee." *Pall Mall Gazette*, July 15, 1868.

General Sabine also commended the Schultz instrument in his presidential address. I made frequent enquiries at Woolwich for some results of experiment made with this chronograph to determine the resistance of the air to the motion of projectiles, but to no purpose. It was just equal to the monster ballistic pendulum. The last news of Schultz's invention that reached me came from the U.S. America, where "it *has been much improved*...and *all* "*the objectionable features* mentioned by the Bashforth Committee "have been obviated. As *thus modified it is strikingly like* " Professor Bashforth's chronograph, and the *same screens, batteries,* " *arrangement of circuits and methods of reduction of observations* "can be used in both"! Ingalls' *Ballistic Machines*, 1885, p. 29. Were these improvements copied from my instrument? What a fall for this *most perfect* instrument!

But Schultz's invention is open to two other fatal objections besides those above mentioned. The cylinder which receives the records is driven by *toothed wheels,* which do not produce a *smooth* motion. And the records are made by *sparks* from a secondary coil, which do not strike *normally* the surface of the cylinder. Either of these defects would be fatal to great precision in experiment. I have determined the errors of the spark records experimentally, and found in one case

+·010; −·003; +·013; ·000; −·004; −·018; +·006; &c.;

and the mean error of 34 records was 0·0107 inch, &c. *Motion of Projectiles* &c., p. 84. And in Fig. 16 (201) I have given a facsimile of the records made by sparks from a point when moving in a straight line.

The Royal Society took great interest in the experiments made with the ballistic pendulum by Robins and Hutton to determine the resistance of the air to the motion of projectiles. So, when I had spent six years of my spare time in successful ballistic experiments, in 1870, I ventured to become a candidate for election by the Royal Society, but in vain. The late Astronomer Royal, Sir G. B. Airy, kindly wrote to me "1870, April 6, "In the list of Candidates for the Royal Society which has just "reached me I see your name. Your certificate has never been "before me, and when I took my round in the Society's Meeting "Room *expressly to look at certificates, I am most confident that* "*it was not there.* Had I seen it I should assuredly have given "myself the pleasure of signing it." I troubled no more about the matter. I left my enemies to enjoy their triumph—such as it was.

In 1871 I published "Tables of Remaining Velocity, Time of [1871
"Flight, and Energy of various Service Projectiles calculated from "the Results of experiments made with the Bashforth Chrono-"graph 1865—1870." Here also were first published General Tables for facilitating the calculation of the *range* corresponding to a given loss of velocity of any *spherical* or *elongated* shot,

$$S_v = S_V - sd^2/w.$$

These tables were reprinted in the *Proceedings of the R.A.* [1871-
Institution with the addition of General Tables for facilitating the calculation of the *time* corresponding to a given loss of *velocity* for any *spherical* or *elongated* shot, 1872, $T_v = T_V - td^2/w$. "These two tables of Mr Bashforth were published in his *Theory* "*of the Motion of Projectiles*, 1872, and they are universally "employed in all our text-books of gunnery," *Woolwich Text Book*, p. 168, 1902.

The Director General of Ordnance resigned his appointment [1872
in 1870, and he was succeeded by a Director of Artillery, who was even less a friend to me and the Advanced Class than those who had preceded him. This was soon made manifest, for in the spring of 1872, when a new class of officers should have been assembled at Woolwich, there were only four candidates for admission, of whom three were disqualified for various reasons. In this dilemma a Committee of three was appointed to consider what was to be done. They found that there was a general desire in the regiment that there should be some scientific education provided. Ultimately it was decided to change the name of the class and to introduce modifications into the old system. *Parliamentary Paper* [C. 589]. This paper is well worth the attention of those who take an interest in these matters.

I published *A Mathematical Treatise on the Motion of Pro-* [1873]
jectiles, founded chiefly on the results of experiments made with the author's chronograph, 1873. Here Bernoulli's famous method of calculating trajectories was adopted. But numerous auxiliary

tables of (X), (Y), and (T) had to be prepared to enable the values of x, y and t to be found readily by calculation.

[1874] March 12, 1874, my particular attention was officially directed to Articles 3 and 4 of a new Royal Warrant of Feb. 12th 1874. As this document indicated a small reduction of my small stipend, I at once retired, for I was not prepared to allow a reduction of a single penny. Considering the *success* of my work, the *labour* it had entailed during the preceding *ten* years, and the *advantage* to the government, a stipend three or four times what I received would not have been too much in return for the services rendered by me to government.

When I retired I supposed that my connection with government and its experiments had come to a close. But it was not so. Major-General F. A. Campbell, R.A., who had been a member of the Committee on the Advanced Class, 1872, became Director of Artillery in 1875. In 1877 I was invited to lend my chronograph and carry out some further experiments for government. I was of course ready to do all in my power to complete the work I commenced thirteen years before. The experiments of 1867, 8 extended from velocity 900 to 1700 f.s. The first object of those renewed experiments was to determine the resistance of the air for velocities below 900 f.s. But afterwards higher velocities were used, and thus the coefficients of resistances to elongated projectiles were determined for velocities 430 to 2250 f.s., by the use of which, corresponding general tables, S_v and T_v were calculated. The *Report* 84/B/2853, July 8, 1879, pp. 58, was at once printed and circulated.

[1880] Immediately afterwards it was decided to make further experiments, by which coefficients of resistance to elongated projectiles were extended to velocities 100 to 2800 f.s. The lower velocities were troublesome to deal with, but I believe that correct results were obtained, for they were carefully tested by the calculation of 32 rounds of the 6·3-inch English Howitzer, and of 82 rounds of the German Kurze 15 cm. Kanone with good results. The results of these experiments of 1880 were combined with those of 1878, 9 and were published together as a Final Report, 84/B/2909, pp. 68, August 31, 1880, without being "referred" to any one, and "with thanks" for the assistance given. By the use of coefficients thus obtained, the resistances in pounds were calculated for diameters 1—20 inches to ogival headed projectiles ($1\frac{1}{2} d$), and also general tables S_v and T_v were prepared for velocities 100 to 2800 f.s.

All these late experiments were successfully carried out by R.A. officers who had passed the course of the Advanced Class, or were superintending experiments at Shoeburyness. The records obtained were sent down to me in the country for reduction. As it had been found by our experiments of 1868 that the resistance of the air varied as the *square* of the velocity for velocities 1300 to 1700 f.s., and also by those of 1880 for velocities 1700 to 2800 f.s. it seems probable the same law, the *Newtonian,* holds good for still higher velocities. This might be tried and settled any day for the highest attainable velocity.

[1881] In 1881 I published a supplement to my *Mathematical Treatise on the Motion of Projectiles,* 1873. Here were given 48 pages of auxiliary tables of (X), (Y) and (T) required to render Bernoulli's solution practically useful. Thus the auxiliary tables were completed for the *cubic* law of resistance.

[1885] In 1885 the sum of £2000 was awarded to me for my experiments by the Marquis of Hartington, who at the same time fully recognised the value of my services to government.

[1890] As the observations made by the help of my chronograph were originally reduced by several persons, it was just possible that some trifling errors might have crept in. I decided therefore to recalculate at my leisure all the good rounds. When that work was completed, I published a Revised Account of my experiments, in 1890, in which I gave new tables of (X), (Y), (T) and (V) to assist in the Bernoulli calculation of trajectories, for the *Newtonian* law of resistance, as well as tables for the *cubic* law of resistance abridged from my former publications. These revised coefficients of resistance differed little from those before published. A supplement to this work was published in 1895, pp. 56, and a second supplement in 1900, pp. 42. The numeration of paragraphs was continuous throughout.

I always had perfect confidence in my results, because every coefficient I admitted was derived from 5 to 10 *consistent records* for each round. I was unwilling to reject any determination, when the round was to all appearance good. Still there are occasionally very unsteady rounds. I have marked with an * those coefficients I felt obliged to reject, and they are only few. And so far as I have had an opportunity of testing my results, the coefficients finally adopted are quite satisfactory. This testing may be carried out by calculating by the use of my general tables, and the horizontal muzzle velocities, the times over 2000 to 5000 yards for the *lowest* elevations given in any range table. This has been done for five guns in my second supplement (340). The following particulars are extracted for example.

Gun	Date	Hor. m.v. f.s.	Range yards	Striking v. f.s.	Time		Diff.
					Range	Calc.	
					″	″	″
4-inch B.L....	1883	1887·97	4000	831·60	10·49	10·589	+0·099
6-inch Q.F....	1891	1875·30	4000	1044·25	8·80	8·816	+0·016
6-inch Q.F....	1893	2145·53	5000	1034·96	10·46	10·452	-0·008
9·2-inch B.L.	1898	2597·92	5000	1629·77	7·27	7·346	+0·076
12-inch B.L.	?	1887·56	4000	1332·10	7·61	7·593	-0·017

Hence it appears that the coefficients of resistance for velocities 2600 to 830 f.s. are correct. And the coefficients for *lower* velocities may be tested by the calculation of trajectories of Howitzers, as has been done in my *Final Report*, pp. 45—47.

But if trajectories be calculated for *high* muzzle velocities and *considerable* elevations, both the calculated *ranges*, and *times* of flight will be found decidedly less than those given in good range tables. The *calculated ranges* may be increased by reducing the coefficients of resistance, but in that case the same coefficients give a *calculated time*, which does not agree with experiment, as Captain May, R.N. found, *Proc. R.A. Inst.* 1886, p. 369. He then properly came to the conclusion that the reduction of the coefficients was not the remedy sought. He therefore justly ascribed the errors to kite-like action.

When an elongated projectile is fired from a rifled gun, it tends to preserve the parallelism of its axis. But when it moves in the direction *ot*, Fig. 1, the resistance of the air, acting in a direction parallel to *to*, causes a pressure on the *under* side greater than that on the *upper* side of the projectile. This will produce a slight upward motion of translation of the projectile, and thus add to the elevation at which the gun was laid. Also the action of the resistance of the air on the head of the projectile tends to push the point of it from the tangent *ot* of the trajectory, and so to cause the axis of the projectile *Ga*, to describe a conical surface about the tangent to the trajectory *Gt*, with a small vertical angle *aGt*, as explained by Magnus. When one quarter of this conical surface has been described by *Ga*, the vertical drift ceases, and the deviation produced afterwards becomes lateral, and both the tangent *Gt* and axis *Ga* are dipping downwards. But if there should afterwards be any drift, upwards or downwards, it would be only slight, and being far from the gun, would not have much effect on the *range* or *time*.

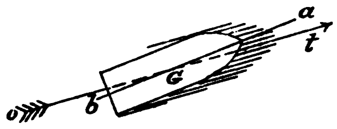


Fig. 1.

As an example calculate the range and time of flight for any elevation 13°, for comparison with the results given in the range table for the 4-inch B.L. gun (1883). We obtain

Calculation,	elevation	13° 0′;	range	5922	yards;	time	18″·152
R. Table	„	13 0;	„	6114	„ ;	„	18 ·955
Difference		0 0	„	−192	„ ;	„	−0·803

so that the calculated *range* and *time* are both decidedly too short.

Next, from the range table find the *elevation* and *time* for a range of 5922 yards,

Calculation,	range	5922	yards;	elevation	13° 0′;	time	18″·152
R. Table	„	5922	„ ;	„	12 20′;	„	18 ·088
Difference		0	„ ;	„	0 40′;	„	0 ·064

so that, if the gun be fired at an elevation of 12° 20′, that elevation is increased, in some way, by 40′, which appears to me to be due to jump and kite-like action. Then the error in *range* is 0, and that in *time* only 0″·064, which is negligible. I have tested in like manner the range table of the 4-inch B.L. gun for elevations 7°, 8°...18°, 19°, and find an increment *in elevation* of 16′ to 87′ required to give both correct *range* and correct *time.*

The effect of this disturbing force will be to add to the elevation at which the gun is laid, an angle, increasing with the elevation till it attains its maximum, and then, as the elevation goes on increasing up to 90°, kite-like action will decrease from its maximum to zero. We have no algebraic expression for this quantity, and it seems hopeless to expect one, as it would be a function of elevation, muzzle velocity, diameter, length, weight, and form of the projectile, and resistance of the air. I regard calculations of this kind, applied to the whole of a range table given in my second supplement, as a strong proof of the accuracy of my *coefficients* of *resistance*, and of my *application* of them to the calculation of trajectories by Bernoulli's solution.

A complaint was made that my course of ballistic experiments was the *most costly* ever known. My reply was that in all 502 rounds of *spherical* and *elongated* projectiles were fired. An average estimate of cost to government might be about £2 per round, thus making £1000. And in return government had obtained results of great value to them. What was the cost of the experiments of Hutton, continued for seven or eight years, which only applied to *spherical* projectiles? What was the object of government in spending several thousand pounds on a monster ballistic pendulum? Was one experiment of the smallest value ever made with that instrument? What was the object in going to this great expense? Was it not a desire to obtain those results which I have supplied for both *spherical* and *elongated* projectiles?

Another complaint has been urged that my work was *theoretical* and not *practical.* In reply to this objection I have only to point out that Krupp has issued a close imitation of my two General Tables, in both French and German, 1881. These tables of mine may be found in the *Handbook for Field Service,* 1879; in the

Manual of Gunnery for H.M. Fleet, 1880; in the *Treatise on Military Small Arms*, 1888; in the *Woolwich Text Book of Gunnery*, 1879, 1883, 1887, 1897 and 1902; and in Noble's *Useful Rules and Tables*, 1882, &c.

My determined opponents at Woolwich were only about three or four in number, but they were very persevering, and they occupied positions which enabled them to be very obstructive in ballistic experiments. I had this opposition to contend against during all the ten years spent by me in H.M. Service. But I do not think the same kind of thing is likely to happen again after what took place in the House of Commons, August 4, 1887.

On the other hand, I enjoyed the thorough support and encouragement of the Council of Military Education, to whom I owed my appointment. I was deeply indebted to Major-General Sir Frederick Abbott, C.B., Member of the Council, and to Lieutenant-Colonel Young, R.A., Director of Artillery Studies, for their advice and assistance in every possible way. Major-General Eardley Wilmot, R.A., who was in command at Shoeburyness when I commenced my experiments, was an earnest and valued supporter of my work.

The War Office did not often interfere, and, so far as I know, they only once acted on the advice of the Ordnance Select Committee when they were in the wrong. I was allowed to make every experiment I desired to make to determine the amount of the resistance of the air to the motion of projectiles. The printing and publishing of the Reports of all our experiments, with the award of the distinguished Referees, were great boons indeed.

I was very much indebted to the Royal Artillery Institution for the publication of a long description of my chronograph, and other papers in their *Proceedings*, as well as for the use of their workshop.

The Director of Artillery Studies, and the Professor of Artillery, R.M.A., were good supporters of my work. I must also thank the three Artillery officers for their assistance in the laborious work of reducing my observations, and other Members of the Advanced Class.

I have also to thank those officers in H.M. Service who have made use of my results in preparing books of instruction for the Navy and the Regiment of the Royal Artillery.

At Shoeburyness, we received every possible assistance, and at times in excess of what was really necessary.

The chronograph by which all my ballistic experiments have been made is now in the Museum, South Kensington. It has been twice borrowed for use at Shoeburyness and Lydd. But if an instrument of the kind should be wanted, it would be best for government to procure one for its own use, and prepare a suitable range. Ten screens, 150 feet apart, were well suited for experiments with 5-inch guns, but perhaps double that distance would be desirable for the largest guns in the service. This chronograph might be used to test all guns for the *steadiness* of their projectiles in the most economical manner. Target practice might be combined with chronographic experiments. As the number of rounds that may be safely fired from heavy guns is very limited, this matter seems worthy of consideration.

NOTE. As auxiliary Tables for a resistance of the air $\propto v^4$ or v^6 by the Bernoulli method of calculating have not yet been prepared, it will suffice to suppose that the resistance to elongated projectiles $\propto v^3$ for velocities 820 to 1050 f.s., where $\log K/g = 0{\cdot}35915$; and also that the resistance $\propto v^3$ for velocities 1050 to 1300 f.s., where $\log K/g = 0{\cdot}53009$.

CONCLUSION.

SOME LATER DISCOVERIES (1872 TO 1900).

My short historical sketch would have ended here had it not been necessary for me to notice the extraordinary proceedings of Mayevski and Krupp since the publication of my reports.

Scientific men have found it necessary to adopt stringent rules to secure to authors the credit of their own published discoveries. So that, when any discovery has been made and published, no other subsequent writer is allowed to claim, or to ignore, that *published* discovery by virtue of his alleged *prior*, yet *unpublished*, discovery of the same results. It is to be hoped that every effort will be made to apply the above necessary rule to all ballistic experiments and discoveries, after the good example set by the editor of *Rivista di Artiglieria e Genio.*

[1872] Mayevski, in his *Traité de Balistique Extérieure*, 1872, noticed the experiments made with *spherical* projectiles by Robins 1742, by Hutton 1787—91, and by Piobert and others at Metz 1839–58. But he was silent respecting my experiments published in 1870. He gave 16 rounds of his Petersburg experiment, then first published, 1872. In Fig. 2, *CD* represents the resistance of the air in pounds to a *spherical* shot, one inch in diameter, for the specified velocities as given in my *Reports* &c., 1870. This also very closely represents the results of Hutton given at p. 4. In the same manner, the centres of the small circles shew the resistances for each of the 16 rounds given by Mayevski in 1872. Thus Mayevski's results, 1872, agreed so closely with my own, 1870, that they contributed nothing new to our knowledge of the resistance of the air to *spherical* projectiles.

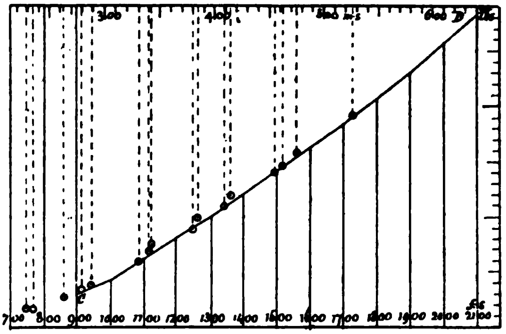


Fig. 2.

In my *Reports* &c., p. 116, there has been given a table of remaining velocities for a *spherical* shot weighing 102 lbs., and 8·9 inches in diameter. For comparison I have recalculated this table using Ingalls' tables, 1886, founded on Mayevski's so-called experiments, 1872.

Distance	400	1400	2400	3400	4400	5400 feet
Bashforth 1870	1970	1680	1437	1236	1078	962 f.s.
Ingalls 1886 Mayevski 1872	1970	1682	1436	1226	1066	950 f.s.
Difference	0	+ 2	− 1	− 10	− 12	− 12

shewing a very slightly *increased* resistance given by the Russian experiments for any velocity below 1400 f.s.

1872] But when Mayevski proceeds to consider "projectiles *oblongs*," he *avowedly copies* 19 *determinations* from my published *Reports* &c., 1870, and professes to contribute 16 determinations of his own for *low* velocities, 1872. In Fig. 3, *AB* represents the resistances of the air in pounds to ogival headed projectiles ($1\frac{1}{2}\,d$) one inch in diameter for the velocities specified in my report, 1870. And, in the same manner, the centres of the small circles give the resistances for the 16 Russian rounds, 1872. These show a resistance a shade greater than that given by my own experiments. The velocities for which Mayevski avowedly copied coefficients from me were 942, 955, 984, 991, 1037, 1050, 1056, 1080, 1096, 1116, 1132, 1165, 1175, 1181, 1375, 1377, 1509, 1667 and 1680 f.s.—enough to give my law of resistance throughout velocities 900 to 1700 f.s. without any Russian experiments whatever.

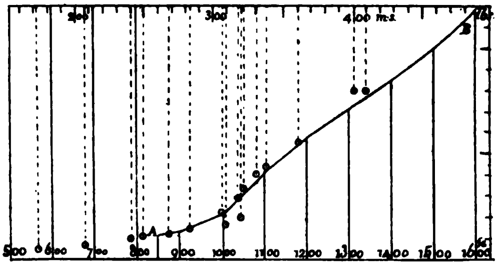


Fig. 3.

Further, Mayevski thought proper to publish the results of this compound as "*russes* et anglaises" where everything of any value was plainly English, and was published 1868 and 1870, or four years before the *Russian discoveries*.

Siacci first published his "Nuovo Metodo per risolvere i problema del Tiro," in *Giornale d' Artiglieria e Genio*, 1880. From the remarks made § 6, it appears that his tables were based on these so called "*Russian* and English" results.

[1872] I will now use column $D(u)$ of these tables to recalculate one of my tables of remaining velocities of ogival headed projectiles, first published in the *Proceedings of the R.A. Institution, short notes,* 1868, p. 70, where a 6·92-inch projectile weighing 115 lbs. was used.

Distance. Feet.	Bashforth 1868. f.s.	Russe ed Inglesi 1872. f.s.	Difference. f.s.
100	1706	1706	0
1100	1603	1605	+ 2
2100	1509	1509	0
3100	1419	1420	+ 1
4100	1336	1336	0
5100	1259	1261	+ 2
6100	1189	1193	+ 4
7100	1129	1134	+ 5
8100	1076	1082	+ 6
9100	1034	1040	+ 6
10100	1002	1005	+ 3

I have not met with the details of a single experiment made by Mayevski to determine the resistance of the air to the motion of a projectile. It is plain that the Russian contributions, 1872, were of no use whatever. Under all these circumstances I decline to recognise Mayevski's experiments as having any independent value whatever.

[1881] In 1881 there were published the "Krupp'sche Tabelle zur "Berechnung der horizontalen Endgeschwindigkeiten und der "Flugzeiten der Langgeschosse" both in German and French, which were close imitations, both in *form* and *matter*, of my *General Tables*, $S_v = S_r - sd^2/w$, and $T_v = T_r - td^2/w$ first published 1871, 2. *AAA*, Fig. 4, represents my law of resistance, 1870, adapted to an ogival head (2 d), and a density of the air such that one cubic metre would weigh 1·206 kil. Also *BBB* gives the law used in Krupp's Tables.

Soon after I obtained a copy of Krupp's Tables, I published a protest against his proceedings in issuing so close an imitation of my work, both in form and substance, without the slightest recognition of my *previous* publication, 1871, 2. This protest was

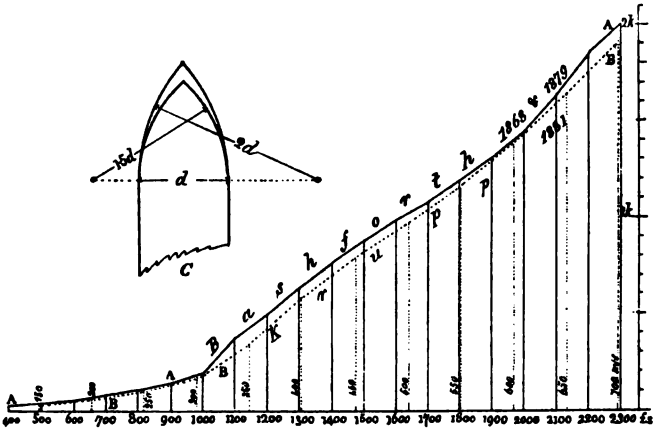


Fig. 4.

[1885] kindly reprinted in the *Rivista di Artiglieria e Genio*, 1885, to which Krupp replied, "Mr Bashforth founds his protest *exclusively* "on the *presumption* that I might have used his publications, "(1) because the resistance of the air taken for the basis of my "tables *did not diverge much* from his own indications, and (2) because my tables as he affirms have appeared since his publication."

I was not aware of these proceedings till many years afterwards, but the editor of the *Rivista* added some very appropriate remarks on Krupp's defence, commencing

"Sic vos non vobis.

"This table except for a small diminution of the resistance due "to a greater acuteness of the Krupp projectile *did not* differ "essentially from that of Mr Bashforth, as one can perceive from "the diagram annexed.

"Of this Mr Bashforth complains, and this does not seem "unreasonable that in the explanations which accompany the "Krupp tables there is *perfect silence* concerning the *English* "*experiments.* In this not only does *every mention of these experi-*

"*ments fail,* but *also all information respecting the experimental* "*foundation* of the tables*." This is a correct statement of the case.

On the publication of Krupp's tables Mayevski made haste to revise his "*Russian* and English" conclusions of 1872, and bring them into accord with those of Krupp, 1881. Although this change is not of the slightest importance to the science of ballistics, it is a very small advantage to me, because the results published by me, 1870, now revert to their rightful English owner without any dispute. [1883

Recently I received from the Adjutant-General's Office, U.S.A., two ponderous volumes of their new ballistic tables. In the introduction Ingalls urges various objections to my experimental results, arising chiefly from erroneous methods of using them. [1900]

Ingalls states that my experiments were made with projectiles *now obsolete. That matters not.* Why not state the simple truth that they were carried out with ogival headed projectiles ($1\frac{1}{2}\,d$), and that the coefficients so obtained are easily adapted to any other useful form of elongated projectile?

My last experiments made to determine the resistance of the air, 1880, stopped at a velocity 2780 f.s. This is not high enough to meet American wants. As I have found that the resistance of the air varies as $(\text{velocity})^2$ for velocities 1300 to 2780 f.s., I shall assume that this, the Newtonian law, holds good for all higher velocities until our government require further experiments to be carried out by my chronograph for higher velocities.

Ingalls confesses that my "early tables have done excellent "service—and with a coefficient of reduction of 0·9 have furnished "*reliable* range tables." That compliment I cannot accept, especially when it is added "except for the longest ranges, for "which, as is *well known,* it is necessary to reduce the coefficient "*very materially below* 0·9, in order to make computation and "observation agree"!! Ingalls' proceedings can never make "com-

---

* See "Replica di Krupp alla protesta del Signor Bashforth" in *Rivista di Artiglieria e Genio,* 1885, or, a translation of the same with notes, Cambridge University Press, 1898. Price 1*s.*

“putation and observation” agree for both *range* and *time*, which are conditions absolutely necessary to be fulfilled.

Sixteen years ago Captain May, R.N. found that calculation, in the usual manner, gave a range shorter than experiment. By reducing the coefficients of resistance he could make calculation and experiment agree for *range*, but his calculated *time* was then *wrong*. He concluded, rightly as I believe, that the observed errors were due to kite-like action, Fig. 1, and were to be therefore corrected by a proper *increase* given to the *elevation* at which the gun was laid, as already explained p. 20. Using the range table of the 4-inch B.L. gun, in my second supplement, I have shewn for each elevation 7° to 19° that a proper increase of the elevation will give by calculation, both a *correct range* and a *correct time* of flight, my coefficients being only reduced three per cent. to adapt them to the form of head of projectile (ogivals 2 *d*) used in the experiment. The only reason for this additional correction is kite-like action, Fig. 1, which was recognised by Didion and others as a “dérivation verticale,” p. 441.

We are told that the American laws of resistance now adopted “are those deduced by General Mayevski from a discussion of the “Krupp firings at Meppen in 1881.” Not so. Mayevski's results are said to be a compound of “les expériences de *Woolwich* (1865–“1870), de Sainte Petersbourg (1868), et de Meppen (1881),” *Revue d'Artillerie*, April 1883, p. 89. From what precedes, it is evident, the American tables owe very much to the Woolwich experiments (1865–1880).

Ingalls has repeated the calculation of one of my nineteen years old examples in his Appendix, p. 234.

Bashforth 1881; $x_0 = 6074$ feet; $t_0 = 10''\cdot413$; $y_0 = 1882$ feet
Ingalls 1900; $x_0 = 6099$ feet; $t_0 = 10\cdot446$; $y_0 = 1885$ feet
Difference + 25 feet + 0·033 + 3 feet

Nothing *obsolete* here.

Again. *Final Report*, p. 45 and Ingalls' Appendix, p. 238,

Bashforth,	1880,	R 3804 yds;	T 21″·60;	striking v. 542·0 f.s.
Ingalls,	1900,	R 3799 yds;	T 21·546;	striking v. 543·2 f.s.
Difference		„ −5 yds	„ −0·054	„ +1·2 f.s.

Nothing *obsolete* here.

But the astounding announcement made by Ingalls is as follows—"As general tables of the definite integrals $^{\phi}T_{\gamma}^{\theta}$, $^{\phi}X_{\gamma}^{\theta}$, "$^{\phi}Y_{\gamma}^{\theta}$, *have not yet been prepared*, the following table was computed "by quadratures for this particular example, *merely to illustrate* "*the method*." Appendix, p. 238. If so, how were my calculations made twenty years ago? The simple fact is that 96 pages in 1873, and 47 pages in 1881, of tables of these quantities, for the *cubic* law of resistance, were published by me. And 68 pages of tables of $^{\phi}X_{\lambda}^{\theta}$, $^{\phi}Y_{\lambda}^{\theta}$, $^{\phi}T_{\lambda}^{\theta}$ and $^{\phi}V_{\lambda}^{\theta}$ for the *Newtonian* law, were published by me in 1890. In all I published 211 pages of these tables which are now said to be non-existent.

In my second supplement I have carefully calculated the *range* and *time* of flight for the 4-inch B.L. gun, for elevations 7°, 8°... 19°, 20°.

Elevation	7°	10°	13°	16°	19°
Calculated Range	4149	5108	5922	6624	7222 yards.
Calculated Time	10″·95	14″·65	18″·15	21″·51	24″·74
Time by R.T.	11·04	14·73	18·09	21·42	24·71
Difference	−0·09	−0·08	+0·06	+0·09	+0·03

These small errors shew that the coefficients of resistance were perfectly good, and also that my adaptation of Bernoulli's method of calculating trajectories is quite satisfactory. But in order to form a Range Table by calculation, it is necessary to make a proper addition to the elevation at which the gun is laid, to allow for jump and kite-like action.

CAMBRIDGE: PRINTED BY J. AND C. F. CLAY, AT THE UNIVERSITY PRESS.

By the same Author.

Description of a Chronograph adapted for Measuring the varying velocity of a body in Motion through the Air, 1866. An extract from the *Proceedings of the R.A. Institution.*

Reports on Experiments made with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Projectiles 1865—1870, pp. 170. London. W. Clowes and Sons. Price 1*s.*

Report on Experiments made with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Elongated Projectiles, Part II., 1878—79. London. Printed for Her Majesty's Stationery Office, 1879.

Final Report of Experiments made with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Elongated Projectiles, 1878—80. London. W. Clowes and Sons, 1880. Price 2*s.* 3*d.*

Tables of Remaining Velocity, Time of Flight and Energy of various Projectiles, calculated from results of Experiments made with the Bashforth Chronograph. London, 1871.

A Mathematical Treatise on the Motion of Projectiles founded chiefly on the Results of Experiments made with the Author's Chronograph. London, 1873.

A Supplement to the above. London, 1881.

A Revised Account of the Experiments made with the Bashforth Chronograph, to find the Resistance of the Air to the Motion of Projectiles, with the Application of the Results to the Calculation of Trajectories, according to J. Bernoulli's Method. Cambridge: at the University Press. 1890. Price 12*s.*

A Supplement to a Revised Account of the Experiments made with the Bashforth Chronograph, &c. 1895. Cambridge: at the University Press. Price 1*s.* 6*d.*

A Second Supplement to a Revised Account of the Experiments made with the Bashforth Chronograph, &c. 1900. Cambridge: at the University Press. Price 1*s.*

Replica di Krupp alla Protesta del Signor Bashforth, translated, with Notes, by FRANCIS BASHFORTH. Cambridge: at the University Press. 1898. Price 1*s.*